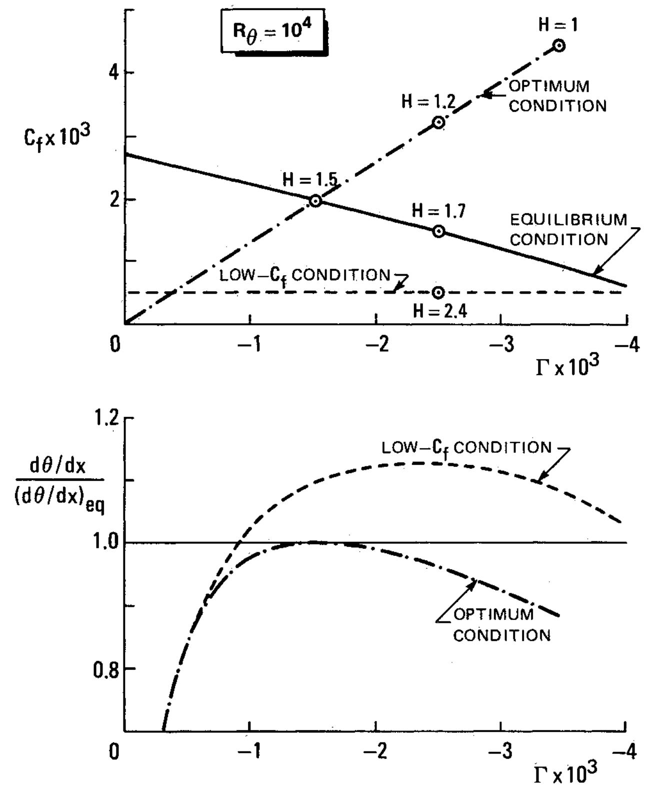

Fig. 1 Boundary layer momentum thickness growth at various conditions.

The corresponding optimum skin friction and minimum momentum thickness growth can also be derived and have been plotted in Fig. 1. The figure also shows the momentum thickness growth, which is obtained when simply aiming at a low skin friction (here $C_f = 0.5 \times 10^{-3}$). The momentum thickness growths are presented relative to the growth in a natural (nonmanipulated) turbulent boundary layer at the same pressure gradient. The skin-friction variation in the natural condition is shown as well. The natural turbulent boundary-layer data have been derived from measurements in nonmanipulated equilibrium turbulent boundary layers.[7]

At zero pressure gradient ($\Gamma = 0$), minimum momentum thickness growth occurs for $C_f = 0$, of course, but with an adverse pressure gradient the minimum appears to be obtained for nonzero skin friction. At larger adverse pressure gradients ($\Gamma < -1.5 \times 10^{-3}$), the skin friction must be increased even above the value in a natural turbulent boundary layer to minimize momentum thickness growth. This occurs at pressure gradients essentially below the gradient leading to flow separation ($\Gamma \approx -4 \times 10^{-3}$ at separation for nonmanipulated boundary layers[8]).

Some shape factor values have been indicated on the curves in Fig. 1. Note that, as essentially $H \geq 1$, the computed optimum curves do not extend beyond $\Gamma \approx -3.5 \times 10^{-3}$. Actually, the accuracy of the underlying skin-friction law, Eq. (3), will probably diminish for $H < 1.2$. Even when taking this into consideration, it is still quite clear from the results that in adverse pressure gradient flows, minimum momentum thickness growth is not obtained when the skin friction is zero, and that fairly high skin frictions may be profitable at larger adverse pressure gradients.

Conclusion

Minimum skin friction does not necessarily mean minimum drag. To minimize drag, account should be taken of both friction and pressure drag. In adverse pressure gradient regions, this means that a low skin friction as well as a small shape factor should be pursued. This restricts the drag reduction potentials, as skin-friction reductions tend to be coupled with shape factor increments. The analysis presented here indicates that at larger adverse pressure gradients this may mean that the skin friction must be increased to reduce the drag.

The general conclusion is that it is not obvious in adverse pressure gradient flows how turbulence must be manipulated to minimize drag. It would be worthwhile to establish theoretically with an optimization method which turbulent shear stress level and distribution would lead at given conditions to minimum drag.

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Applications of Various Coordinate Transformations for Rotating Disk Flow Stability

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Introduction

It has already been established[1] that the instability mechanism of the rotating disk flow is similar to that of other three-dimensional boundary layers. The analytical studies of the rotating disk flow have commonly benefited from existence of the basic flow based on von Karman's[2] similarity solution. Discrepancies between earlier analyses and the experiments[1,3] for the critical Reynolds number were reduced in more recent works[4,5] by including the Coriolis force and streamline curvature. The resulting equations are, however, sixth-order instead of the fourth-order Orr-Sommerfeld equation.

In the present work the linear, temporal stability of the rotating disk flow is investigated by a spectral method based on Chebyshev polynomials with emphasis on the various coordinate transformations.

Formulation

A flat disk of infinite radius rotating about the vertical axis with angular velocity Ω is considered. Applying von Karman's

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Table 1 Comparison of principal eigenvalues ($R=280$, $\alpha=0.395$, $\beta=0.1$)

J	Algebraic tranformation I	Algebraic transformation II	Exponential tranformations
20	$-1.13389\ E-2+i\ 2.08222\ E-4$	$-1.11216\ E-2-i\ 1.95667\ E-4$	$-1.11769\ E-2-i\ 1.95421\ E-4$
30	$-1.11792\ E-2-i\ 1.97737\ E-4$	$-1.17762\ E-2-i\ 1.92526\ E-4$	$-1.11763\ E-2-i\ 1.92538\ E-4$
40	$-1.11763\ E-2-i\ 1.92576\ E-4$	$-1.11763\ E-2-i\ 1.92538\ E-4$	$-1.11763\ E-2-i\ 1.92538\ E-4$
50	$-1.11763\ E-2-i\ 1.92538\ E-4$	$-1.11763\ E-2-i\ 1.92538\ E-4$	$-1.11763\ E-2-i\ 1.92538\ E-4$
60	$-1.11763\ E-2-i\ 1.92438\ E-4$	$-1.11763\ E-2-i\ 1.92538\ E-4$	$-1.11763\ E-2-i\ 1.92538\ E-4$

similarity relations on the governing equations yields a system of four nonlinear equations for the similarity variables F, G, H, P, corresponding to velocity components and pressure, respectively. The derivatives of F and G at the surface are evaluated by using a Regula-Falsi[6] algorithm with a very small tolerance for convergence in order to improve accuracy of stability analysis. Next, unsteady three-dimensional disturbances are superposed on the basic flow variables and the normal modes approach is used. By retaining the terms up to the order $1/R$, one obtains a sixth-order system of equations for axial perturbation velocity ϕ, and vorticity η, similar to the one given in Malik, et al.[4] except for terms corresponding to the fixed coordinate system used in this work.

The dependent variables are expressed as

$$\phi=\Sigma a_n f_n(z)$$

and

$$\eta=\Sigma b_n f_n(z)$$

where a_n and b_n are the complex coefficients and f_n is the Chebyshev polynomial of order n. In general, a complete system is obtained by satisfying the differential equations at discrete collocation points, in addition to appropriate boundary conditions. Derivatives of Chebyshev polynomials are evaluated by using recursion relationships given in Fox and Parker.[7] In the process, one needs the basic flow solutions at discrete, unevenly placed collocation points. This is achieved by modifying the Runga-Kutta integrator accordingly. The result is the generalized algebraic eigenvalue problem

$$[A]\{x\}=\omega[B]\{x\}$$

for the complex coefficients a_n and b_n. This is manipulated to a standard eigenvalue problem, keeping in mind that $[B]$ is a singular matrix due to the difference in the orders of differentiation of ϕ and η for the global solution of the eigenvalue problem.

Because the physical coordinate is in semi-infinite space, $(0\le z\le\infty)$, one needs a coordinate transformation of mapping the physical space to the Chebyshev spectral space $|\xi|\le 1$. In this study two algebraic transformations and an exponential transformation are used and results compared.

Algebraic Tranformations

The first algebraic transformation is:

$$\xi=(z-z_o)/(z+z_o) \tag{1}$$

This transformation is studied by Grosch and Orszag[8] and used in Malik et al.[4] It maps the physical space to $-1\le\xi\le 1$ and requires use of both even and odd Chebyshev polynomials. Following Ref. 8, the scale factor z_o is taken as 1.8 and is found to be satisfactory. The consideration of redundancy (A being a singular matrix), dictates the series expansions to be

$$\phi_j=\sum_{n=o}^{J+1} a_n T_n(\xi_j)$$

and

$$\eta_j=\sum_{n=0}^{J} b_n T_n(\xi_j)$$

This will introduce a complete system of $(2J+3)$ in size where J is the number of collocation points. The spectral coordinate is defined by

$$\xi_j=\cos(\Pi_j/J)$$

The boundary conditions involving derivatives at infinity are dropped and each equation is satisfied at $(J-1)$ collocation points.

As a second transformation

$$\xi=z_o/(z+z_o) \tag{2}$$

is used. It limits the transformed space to $0\le\xi\le 1$ and only odd Chebyshev polynomials are needed. This transformation is due to Spalart,[9] whose scale factor of $z_o=4.5$ is adopted here. As pointed out by Spalart, use of odd polynomials results in concentration of collocation points closer to the wall. Therefore, one would expect better results from this transformation for the same number of collocation points. The series expansions are the same as in the previous transformation. All boundary conditions at infinity are dropped, but equations are satisfied at all collocation points; thereby a complete system is obtained.

Exponential Transformation

An exponential transformation is used for mapping in the form of

$$\xi=\exp(-z/z_o) \tag{3}$$

This transformation also maps $0\le z\le\infty$ to $0\le\xi\le 1$ and requires odd polynomials. A parametric study is made for a suitable value of scale factor and $z_o=6.5$ is found to be satisfactory.

A sample of comparisons between the three different transformations is shown in Table 1 for values of $\alpha=0.395$, $\beta=0.1$, and $R=280$ where α and β are wave numbers in axial and azimuthal directions, respectively, and the Reynolds number is

$$R=r_e\sqrt{\Omega/\nu}$$

where r_e is the radial location near which the analysis is made. The number of collocation points is varied from 20 to 60 and the principal eigenvalues are given. It appears that the exponential transformation yields the correct eigenvalue with the smallest number of collocation points. The results, however, are not conclusive since in some cases increasing J resulted in variations in the converged values. This is primarily due to inconsistency of boundary conditions with asymptotic behavior of the solutions, as explained in Ref. 8 (p. 284). Another explanation is that the value of the collocation point corresponding to infinity shows a large variation by J in the exponential transformations. On the other hand, the second algebraic transformation is consistently

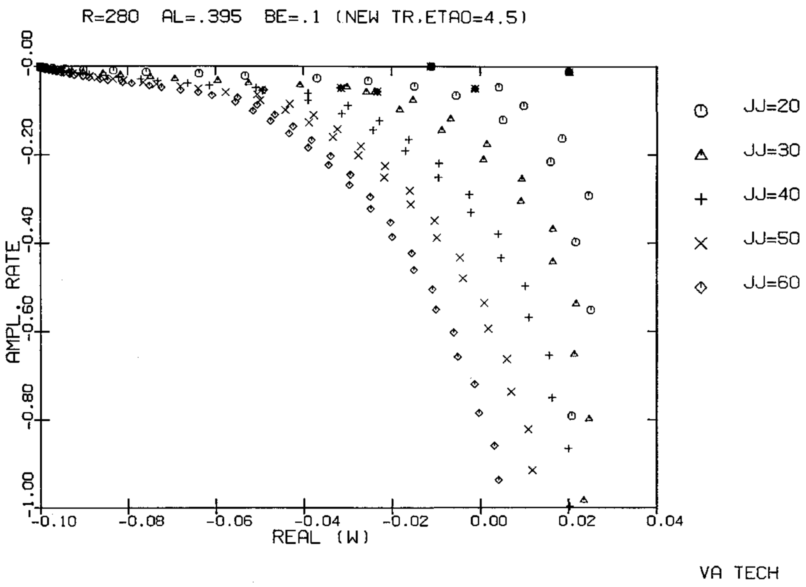

Fig. 1 Eigenvalue spectra for the increasing number of total collocation points: $R=280$, $\alpha=0.395$, $\beta=0.1$.

better than the other transformation, and does not have any convergence problem. Therefore, it is more accurate and useful than the other two transformations considered here.

In this paper, the critical Reynolds number is sought. Therefore, only the principal eigenvalue is of interest. For this purpose, a local search method is employed by using start-up values from the global computations. Details of this process are given by Sahin.[10]

Results and Conclusions

The critical Reynolds number for the rotating disk flow is investigated by using a Chebyshev spectral method. Two algebraic and one exponential transformations are used for mapping the physical space to the spectral space. One of the algebraic transformations is found to be best among the three, partially due to having a resolution of collocation points closer to the wall. Although it appears very efficient in some cases, the exponential transformation is found to have the problem of inconsistency.

In Fig. 1, the effect of J on the eigenvalue spectrum is demonstrated. Increasing J causes more discrete eigenvalues separate from the continuous spectrum. Apparent pairing of the eigenvalues is due to having a system of two differential equations.

By using a local search method, the critical Reynolds number (R_c) is found to be 285, as explained in Ref. 10. This result is consistent with the result of Ref. 4 ($R_c=287$) and in the more recent work of Malik[11] ($R_c=285.36$) that is obtained by a Newton-type iteration. As indicated,[11] this is in excellent agreement with experiments of Wilkinson and Malik.[12] The experiments, however, are not based on controlled normal-mode-type disturbances. Despite success of the linear theory, discrepancies, such as number of vortices observed, still exist between the experiment.

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Supersonic Base Pressure and Lipshock

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Introduction

MANY theories have been proposed to describe base flows. Probably the best known theory is the Chapman-Korst[1] model. This model and its subsequent derivatives are

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